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Section 2.2 Zero and Negative Exponents

Name: Alice Zhao Date: Oct 28th, 2010

1. Simplify each of the following expressions without a calculator:

a) 5^{-1}

$\frac{1}{5}$ ✓

b) $(-5)^{-2}$

$=\frac{1}{(-5)^2}=\frac{1}{25}$

$\frac{1}{25}$ ✓

c) 223^0

1 ✓

d) $(0.2)^{-3}$

$=\frac{1}{(\frac{1}{5})^3}=1\cdot 5^3=125$

125 ✓

e) $(0.5)^{-5}$

$=\frac{1}{(\frac{1}{2})^5}=1\cdot 2^5=32$

32 ✓

f) $(-a)^{-3}$

$=\frac{1}{(-a)^3}$

$\frac{1}{(-a)^3}$ ✓

g) $(-3b)^{-3}$

$=\frac{1}{(-3b)^3}=\frac{1}{-27b^3}$

$\frac{1}{-27b^3}$ ✓

h) $(4a^3b^2)^{-2}$

$=\frac{1}{(4a^3b^2)^2}=\frac{1}{16a^6b^4}$

$\frac{1}{16a^6b^4}$ ✓

i) $-(-2a^{-1})^{-4}$

$=-\left(\frac{1}{-2a}\right)^{-4}=-\left(1\cdot(-2a)^4\right)$

$-16a^4$ ✗ $=-16a^4$

j) $-(-3ab^{-1})^{-3}$

$=-(-3ab^3)=-(-27a^3b^3)$

$27(-a^3)(-b^3)$ ✗

2. Express each of the following as a common fraction:

a) $\left(\frac{2}{3}\right)^{-2}$

$=\frac{1}{(\frac{2}{3})^2}=\frac{1}{\frac{4}{9}}=\frac{9}{4}$

$\frac{9}{4}$ ✓

b) $\left(-\frac{4}{3}\right)^{-2}$

$=\frac{1}{(-\frac{4}{3})^2}=\frac{1}{\frac{16}{9}}=\frac{9}{16}$

$\frac{9}{16}$ ✓

c) $\left(\frac{1}{3}\right)^{-3}$

$=\frac{1}{(\frac{1}{3})^3}=3^3=27$

27 ✓

d) $\left(\frac{2}{3}+\frac{3}{2}\right)^{-2}$

$=\left(\frac{4}{6}+\frac{9}{6}\right)^{-2}=\frac{1}{(\frac{13}{6})^2}$

$\frac{36}{169}$ ✓ $=\frac{36}{169}$

e) $\left(\frac{1}{2}+\left(\frac{1}{3}\right)^{-1}\right)^{-2}$

$=\left(\frac{1}{2}+3\right)^{-2}=\frac{1}{(\frac{7}{2})^2}$

$\frac{4}{49}$ ✓ $=\frac{4}{49}$

f) $\left(\frac{1}{6}-\frac{1}{12}\right)^{-2}$

$\left(\frac{1}{12}\right)^{-2}=\frac{1}{(\frac{1}{12})^2}=144$

144 ✓

g) $(5^{-2}-5^{-1})^{-2}$

$\left(\frac{1}{25}-\frac{5}{25}\right)^{-2}=\left(-\frac{4}{25}\right)^{-2}$

$=\frac{1}{(-\frac{4}{25})^2}=\frac{25^2}{4^2}=\frac{625}{16}$

$\frac{625}{16}$ ✓

h) $-\left(\frac{-a}{b}\right)^{-4}$

$=-\frac{1}{(\frac{-a}{b})^4}=-\frac{b^4}{(-a)^4}$

$-\frac{b^4}{a^4}$ $=-\frac{b^4}{a^4}$ ✓

i) $\left(\frac{b}{a}-\frac{a}{b}\right)^{-1}$

$=\frac{1}{\frac{b^2-a^2}{ab}}=\frac{ab}{b^2-a^2}$

$\frac{ab}{a^2-b^2}$ ✗

j) $\left(\frac{a}{b}-\frac{b}{a}\right)^{-2}$

$=\left(\frac{a^2}{ab}-\frac{b^2}{ab}\right)^{-2}=\left(\frac{ab}{a^2-b^2}\right)^2$

$\left(\frac{ab}{a^2-b^2}\right)^2$ ✗

3. Express in simplist form:

a) $\dfrac{3^{-2}+4^{-1}}{5^{-1}}$

$=\dfrac{\frac{1}{3^2}+\frac{1}{4}}{\frac{1}{5}}=\frac{13}{180}$

$\frac{13}{180}$ ✗

b) $\dfrac{2(3)^{-2}+3(2)^{-3}}{2(3)^{-2}-3(2)^{-3}}$

$=\dfrac{\frac{2}{9}+\frac{3}{8}}{\frac{2}{9}-\frac{3}{8}}=\dfrac{\frac{43}{72}}{\frac{-11}{72}}=\frac{43}{72}\cdot\frac{72}{-11}$

$-\frac{43}{11}$ ✓

c) $\dfrac{(9-6)!\;+\;3^{-2}}{(10-7)!}$

$\dfrac{6+\frac{1}{9}}{6}=\frac{6}{9}\times\frac{1}{6}$

$\frac{1}{9}$ ✗

d) $\left[4-3(6-8)^{-1}\right]^{-2}$

$=\left[4-3\left(\frac{1}{-2}\right)\right]^{-2}$

$=\left(4+\frac{3}{2}\right)^{-2}=\left(\frac{11}{2}\right)^{-2}$

$=\left(\frac{2}{11}\right)^2=\frac{4}{121}$ ✓

$\frac{4}{121}$

e) $\dfrac{5^{-2}\times 4^{-2}}{20^{-2}}$

$=\frac{20^2}{5^2 4^2}=\frac{20}{5\cdot 4}=1$

1 ✓

f) $\left(\dfrac{1}{ab^2}\right)^{-3}$

$=(ab^2)^3=a^3b^6$

a^3b^6 ✓

g) $\left(\dfrac{-2x^2}{3y}\right)^{-3}\div\left(\dfrac{3}{x}\right)$

$\left(\frac{3y}{-2x^2}\right)^3\cdot\frac{x}{3}=\frac{3^3y^3}{-8x^6}\cdot\frac{x}{3}$

$=\frac{3^2y^3}{-8x^5}=\frac{9y^3}{-8x^5}$

$\frac{9y^3}{-8x^5}$ ✓

h) $\dfrac{ab^{-2}+ba^{-1}}{ab^{-3}}$

$=b+a^{-2}b^4$

$=b+\frac{b^4}{a^2}$

$b+\frac{b^4}{a^2}$ ✓

i) $(ab)^{-2}+b^{-2}$

$=\frac{1}{(ab)^2}+\frac{1}{b^2}$

$=\frac{1}{a^2b^2}+\frac{a^2}{a^2b^2}$

$=\frac{a^2}{a^2b^2}=\frac{1}{b^2}$

$\frac{1}{b^2}$ ✗

j) $\dfrac{a^{-3}+b^{-3}}{ab^{-3}}$

$=\dfrac{\frac{1}{a^3}+\frac{1}{b^3}}{\frac{a}{b^3}}=\dfrac{\frac{b^3+a^3}{a^3b^3}}{\frac{a}{b^3}}$

$=\frac{b^3+a^3}{a^4}=\frac{b^3}{a^4}+\frac{1}{a}$

$\frac{b^3}{a^4}+\frac{1}{a}$ ✗

4. Solve for 'x' in each of the following equations:

a) $\left(\frac{1}{3^4}\right)^{-2} = 9^x$

$(3^4)^2 = 9^x$

$x = 4$

b) $\left(\frac{-1}{2}\right)^{-5} = (-2)^x$

$(-2)^5 = (-2)^x$

$x = 5$

c) $\left(\frac{1}{4^{-2}}\right)^{-3} = 16^x$

$(4^{-2})^3 = (4^2)^{-3}$

$x = -3$

d) $\left(\frac{1}{4}\right)^{-2} = 8^{-x}$

$4^2 = 2^4 = 2^{-3x}$ $\quad x = -\frac{4}{3}$

$x = -\frac{4}{3}$

e) $(3^{14})^{-2} = \left(\frac{1}{9}\right)^x$

$\frac{1}{3^{28}} = \frac{1}{3^{2x}}$ $\quad x = \frac{28}{2} = 14$

$x = 14$

f) $\left(\frac{3^8}{2^4}\right)^{x-2} = 1$

$x - 2 = 0 \quad x = 2$

$x = 2$

g) $2^{-x} + 2^{-x} = 8$

$= \frac{1}{2^x} + \frac{1}{2^x} = \frac{2}{2^x} = 8$

$= 2^{x-1} = 2^3$

$x = 4$ ✗

h) $(5)^{-2} = (125)^x$

$(5)^{-2} = 5^{3x}$

$x = -\frac{2}{3}$

$x = -\frac{2}{3}$

i) $2^{-3} - 8^{-1} = 2^x$

$\frac{1}{8} - \frac{1}{8} = 2^x$

$0 = 2^x$

No solutions

j) $3^{-3} - 3^{-2} = 3^x$

$\frac{1}{27} - \frac{3}{27} = 3^x$

$\frac{-2}{27} = 3^x$ (will never be negative)

No solutions

5. List the following from the least to the greatest:

i) $\left(\frac{1}{8}\right)^{-3}$, ii) $\left((-2)^2\right)^3$, iii) $\left(-\frac{1}{2}\right)^{-2}(-2)^{-2}$, iv) $(-2)^{-2}\left(\left((2)^{-1}\right)^{-2}\right)^{-3}$ $\quad$ iv < iii < ii < i

$8^3 = 512$ $\quad 4^3 = 64$ $\quad \frac{1}{(-\frac{1}{2})^2} \cdot \frac{1}{(-2)^2} = \frac{1}{\frac{1}{4} \cdot 4} = 1$ $\quad \frac{1}{(-2)^2} \cdot 2^{-6} = \frac{1}{4} \cdot \frac{1}{2^6} = \frac{1}{4 \cdot 64}$

$\frac{1}{256} < 1 < 64 < 512 \Rightarrow$ iv) < iii) < ii) < i) $\quad = \frac{1}{256}$

6. If $\frac{1}{2x+1} = 3^{-1}$, then what is the value of $2x+8$?

$\frac{1}{2x+1} = \frac{1}{3}$ $\quad 2x + 1 = 3$ $\quad 2x = 2$ $\quad x = 1$

$2(1) + 8 = \boxed{10}$

7. If $4^{-3} + 4^{-3} + 4^{-3} + 4^{-3} = 2^x$, then what is the value of 'x'?

$4^{-3} + 4^{-3} + 4^{-3} + 4^{-3} = 4^{-3+1} = 2^x$

$4^{-2} = 2^{2 \cdot -2} = 2^{-4} = 2^x$ $\quad \boxed{x = -4}$

8. What is the reciprocal of $\left(\frac{2}{3} + \frac{3}{2}\right)^{-3}$. Express your answer as a common fraction.

$\left(\frac{4}{6} + \frac{9}{6}\right)^{-3} = \frac{1}{\left(\frac{13}{6}\right)^3} = \left(\frac{6}{13}\right)^3 = \boxed{\frac{216}{2197}}$ ✗

9. Express $\left(1^{-1}+2^{-1}+3^{-1}+4^{-1}+5^{-1}+6^{-1}\right)^{-1}$ as a common fraction in simplist term.

$\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)^{-1} = \frac{1}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}} = 1+2+3+4+5+6 = \boxed{21}$

X

10. Solve for x: $2+\left(2+\left(2+(2)^{-1}\right)^{-1}\right)^{-1}=x$. Express your answer as a common fraction.

$2+\left(2+\left(2+\frac{1}{2}\right)^{-1}\right)^{-1} = 2+\left(2+\left(\frac{5}{2}\right)^{-1}\right)^{-1} = 2+\left(2+\frac{2}{5}\right)^{-1} = 2+\left(\frac{12}{5}\right)^{-1} = 2+\frac{5}{12} = \boxed{\frac{29}{12}}$

✓

11. Given that the positive integers 'a','b',and 'c' satisfy the equation: $\frac{4}{5}=a^{-1}+b^{-1}+c^{-1}$. What is the largest value of $a+b+c$?

$\frac{4}{5}=\frac{8}{10}=\frac{12}{15}=\frac{16}{20}=\frac{20}{25}=\frac{24}{30}$

$\frac{4}{5}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{2}+\frac{1}{20}+\frac{1}{4}$ ← sum of denominator: $2+20+4=\boxed{26}$

✓

20 = 1, 2, 4, 5, 10 $\quad \frac{16}{20}=\frac{10}{20}+\frac{2}{20}+\frac{4}{20}=\frac{1}{2}+\frac{1}{10}+\frac{1}{5}$

10, 2, 4

10, 1, 5 $\quad =\frac{10}{20}+\frac{1}{20}+\frac{5}{20}=\frac{1}{2}+\frac{1}{20}+\frac{1}{4}$

12. For what values of "x" will the expression be true? $4\left(4^2+4^2+4^2+4^2\right)=2^x$

$4(4^2+4^2+4^2+4^2)=2^x$

$4(4^1\cdot 4^2)=4(4^3)=4^4=2^8=2^x$ $\quad \boxed{x=8}$

✓

13. Jason entered a positive number onto his calculator and pressed the x^2 button and got the value $0.\overline{012345679}$. What value would he have gotten if he had pressed the $\sqrt{x}$ button instead? Try to do this question without a calculator. $0.\overline{012345679}=\frac{12345679}{999999999}=x^2$

$\sqrt{x}=\sqrt[4]{\frac{12345679}{999999999}}=\sqrt[4]{\frac{1}{81}}=\boxed{\frac{1}{3}}$

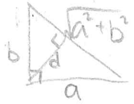
✓

14. Challenge: The posistive integers a,b,and c satisfy $a^{-2}+b^{-2}=c^{-2}$. What is the sum of all possible values of "a" such that $a\le 100$?

$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

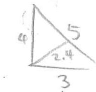
$\frac{b^2}{a^2b^2}+\frac{a^2}{a^2b^2}=\frac{1}{c^2}$ → $\frac{a^2b^2}{a^2+b^2}=c^2$

$c=\frac{ab}{\sqrt{a^2+b^2}}$



$a\cdot b=d\cdot\sqrt{a^2+b^2}$

$d=\frac{a\cdot b}{\sqrt{a^2+b^2}}$



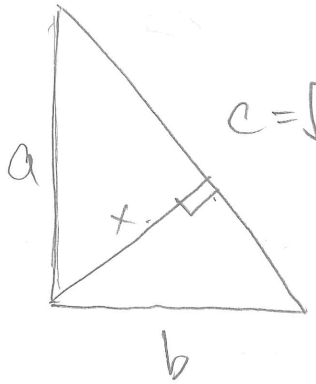
X = doesn't work

∴ let's times all by 5

15, 30, 45, 60, 75, 90 $\quad$ $\boxed{\text{11 possibilities}}$

20, 40, 60, 80, 100

X

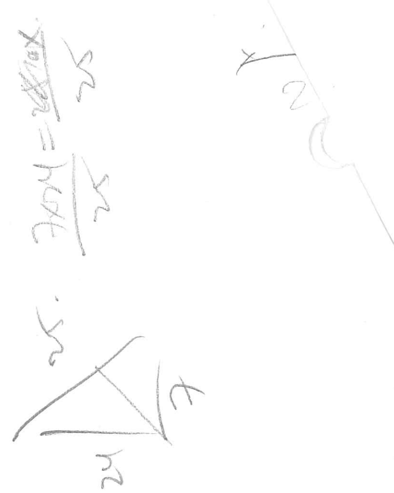


$c = \sqrt{a^2+b^2}$

① $xc = ab$.

$x = \frac{ab}{c}$

$\boxed{x = \frac{ab}{\sqrt{a^2+b^2}}}$ //



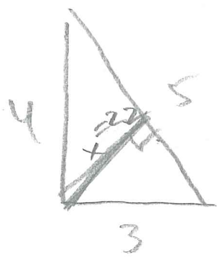
② $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

$\frac{b^2+a^2}{a^2b^2} = \frac{1}{c^2}$

$\frac{a^2b^2}{b^2+a^2} = c^2$

$\frac{ab}{\sqrt{a^2+b^2}} = c$

③. $\boxed{x = c}$ //



$x = \frac{12}{5} = 2.2$

∴ if $a=3$ $b=4$. $c=2.4$

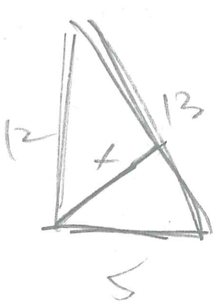
×5

$\boxed{a=15}$ $b=20$ $\boxed{c=12}$.

×10.

$\boxed{a=30}$ $b=40$ $c=22$

×15



$5 \times 12 = 13x$

$\frac{60}{13} = x$

a b c
5, 12, $\frac{60}{13}$

65 12×13, 60.

×5	a = 15, 20	
×10	a = 30, 40	
×15	a = $\boxed{45}$, $\cancel{\boxed{60}}$	12
×20	= 60, 80	
×25	75, 100	$x = 55$ 300
×30	90, +	

11 65

×120 $\frac{1}{45^2} + \frac{1}{60^2} = \frac{1}{12^2}$

$\frac{1}{2025} + \frac{1}{3600} + \frac{1}{1089}$

5

20

0.000771604

620.

Blk: 3

SECTION 2.2 CORRECTIONS

1. i) $-(-2a^{-1})^{-4}$

$= -\left(-2\frac{1}{a}\right)^{-4}$

$= -\left(-\frac{2}{a}\right)^{-4}$

$= -\left(\frac{1}{\left(-\frac{2}{a}\right)^4}\right) = -\left(\frac{1}{\frac{16}{a^4}}\right)$

$= -\left(\frac{a^4}{16}\right) = \boxed{-\frac{a^4}{16}}$

j) $-(-3ab^{-1})^{-3}$

$= -\left(-3\frac{a}{b}\right)^{-3}$

$= -\left(-\frac{3a}{b}\right)^{-3} = -\left(\frac{1}{\left(-\frac{3a}{b}\right)^3}\right)$

$= -\left(\frac{1}{-\frac{27a^3}{b^3}}\right) = -\left(-\frac{b^3}{27a^3}\right)$

$= \boxed{\frac{b^3}{27a^3}}$

2. i) $\left(\frac{b}{a} - \frac{a}{b}\right)^{-1}$

$= \left(\frac{b^2-a^2}{ab}\right)^{-1}$

$= \boxed{\frac{ab}{b^2-a^2}}$

j) $\left(\frac{a}{b} - \frac{b}{a}\right)^{-2}$

$= \left(\frac{a^2-b^2}{ab}\right)^{-2}$

$= \left(\frac{ab}{a^2-b^2}\right)^{2}$

3. a) $\frac{3^{-2}+4^{-1}}{5^{-1}}$

$= \frac{\frac{1}{9}+\frac{1}{4}}{\frac{1}{5}} = \frac{13}{36} \cdot 5 = \boxed{\frac{65}{36}}$

c) $\frac{(9-6)! + 3^{-2}}{(10-7)!}$

$= \frac{3! + 3^{-2}}{3!} = \frac{6+\frac{1}{9}}{6} = \frac{55}{9} \times \frac{1}{6} = \boxed{\frac{55}{54}}$

i) $(ab)^{-2} + b^{-2}$

$= \frac{1}{(ab)^2} + \frac{1}{b^2}$

$= \frac{1}{a^2b^2} + \frac{1}{b^2}$

$= \boxed{\frac{1+a^2}{a^2+b^2}}$

j) $\frac{a^{-3}+b^{-3}}{ab^{-3}}$

$= \frac{\frac{1}{a^3}+\frac{1}{b^3}}{\frac{a}{b^3}} = \frac{\frac{b^3+a^3}{a^3b^3}}{\frac{a}{b^3}} = \frac{b^3+a^3}{a^3b^3} \cdot \frac{b^3}{a} = \boxed{\frac{b^3+a^3}{a^4}}$

4. g) $2^{-x} + 2^{-x} = 8$

$2^1 \cdot 2^{-x} = 2^3$

$2^{1-x} = 2^3$

$1-x = 3 \quad x = 1-3 = \boxed{-2}$

$\left(\frac{a^2-b^2}{ab}\right)^{-2}$

$\frac{(ab)^2}{(a^2-b^2)^2} \cdot \cancel{= \frac{ab^2}{(a^2-b^2)(a^2-b^2)}}$

8. Reciprocal of $\left(\frac{2}{3}+\frac{3}{2}\right)^{-3}$

$\left(\frac{4}{6}+\frac{9}{6}\right)^{-3} = \left(\frac{13}{6}\right)^{-3} = \left(\frac{6}{13}\right)^3 = \frac{216}{2197}$ $\boxed{\frac{2197}{216}}$

9. $(1^{-1}+2^{-1}+3^{-1}+4^{-1}+5^{-1}+6^{-1})^{-1}$

$= \left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)^{-1}$

$= \left(\frac{60}{60}+\frac{30}{60}+\frac{20}{60}+\frac{15}{60}+\frac{12}{60}+\frac{10}{60}\right)^{-1}$

$= \left(\frac{147}{60}\right)^{-1} = \frac{60}{147} = \boxed{\frac{20}{49}}$

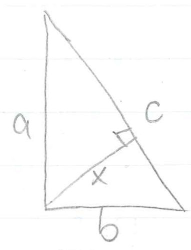
14. $a^{-2}+b^{-2}=c^{-2}$

$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

$\frac{b^2}{a^2b^2}+\frac{a^2}{a^2b^2}=\frac{1}{c^2}$

$\frac{a^2b^2}{a^2+b^2}=c^2$

$\boxed{c=\frac{ab}{\sqrt{a^2+b^2}}}$

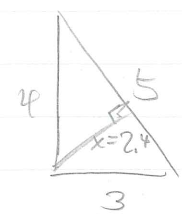


$xc=ab$

$x=\frac{ab}{c}$

$\boxed{x=\frac{ab}{\sqrt{a^2+b^2}}}$

$\boxed{x=c}$



∴ if $a=3$ $b=4$ $c=2.4$

×5

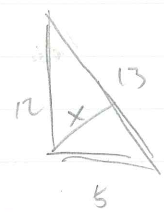
$a=15$ $b=20$ $c=12$

×10

$a=30$ $b=40$ $c=24$

×15

⋮



$5\times 12=13x$

$\frac{60}{13}=x$

	a	b	c
	5	12	$\frac{60}{13}$
×13	65	156	60

×5 a = 15, 20

×10 a = 30, 40

×15 = 45, ~~60~~ ← repition

×20 = 60, 80

×25 = 75, 100

×30 = 90 X

65 } 11 possibilities

Sum: $15+20+30+40+45+60+80+75+100+90+65=\boxed{620}$